A New Approach to Image Denoising based on Diffusion-MLP-LMMSE

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Abstract: In this paper, diffusion wavelet-based multiscale linear minimum mean square-error estimation (LMMSE) scheme for image denoising in conjunction to neural network is proposed, and the determination of the optimal wavelet basis with respect to the proposed scheme is also discussed. Generally, the over complete wavelet expansion (OWE) is more effective than the orthogonal wavelet transform (OWT) specially in image noise reduction problems. For exploring the strong interscale dependencies of OWE, we combine the pixels at the same spatial location across scales as a vector and apply LMMSE to the vector. Compared with the LMMSE within each scale, the interscale model exploits the dependency information distributed at adjacent scales. The performance of the proposed scheme is dependent on the selection of the wavelet bases. The optimal wavelet that achieves the best tradeoffs between the two criteria can be determined from a library of wavelet bases. To estimate the wavelet coefficient statistics precisely and more accurately, we use the MLP approach of neural network which exploits the wavelet intrascale dependency and yields a local discrimination of images. The scheme improves the denoising rate as the training images are increased.

Keywords: Diffusion Wavelet, Diffusion Packets, OWE, MLP, OWT

Introduction

Images are frequently corrupted with noise during acquisition, transmission, and retrieval from storage media. Noise corrupts both images and videos [1]. The purpose of the denoising algorithm is to remove such noise. In addition, some fine details in the image may be confused with the noise or vice-versa. Many image-processing algorithms such as pattern recognition need a clean image to work effectively. Images are affected by different types of noise. Denoising of natural images corrupted by noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. The wavelet denoising scheme thresholds the wavelet coefficients arising from the wavelet transform. Problem of denoising can be formulated as [2]: Let A(i,j) be the noise free image and B(i,j) be the image corrupted with noise Z(i,j) then

\[ B(i,j) = A(i,j) + \sigma Z(i,j) \]  \hspace{1cm} (1)

where \( \sigma \) is the noise variance. In the wavelet domain, the problem can be formulated as

\[ Y(i,j) = X(i,j) + W(i,j) \]  \hspace{1cm} (2)

Where Y(i,j) is the noisy wavelet coefficient X(i,j) is the true coefficient and W(i,j) is noise. Since the first wavelet soft thresholding approach of Donoho [3], many wavelet-based denoising schemes were reported [4]–[5], [6]–[7], [8], [9],[10]. In all these approaches different wavelets are applied and accordingly wavelet coefficients are computed in a formal way.

In this paper diffusion wavelet-based multiscale linear minimum mean square-error estimation (LMMSE) scheme is applied. In this scheme as a first step a diffusion wavelet is applied on the image which is to be denoised. Although WT well decorrelates signals, strong intrascale and interscale dependencies between wavelet coefficients may still exist. If a coefficient at a coarser scale has small magnitude, its descendants at finer scales are very likely to be small too. Shapiro[11] exploited this property and developed the well-known embedded zero tree wavelet image compression scheme. In another viewpoint, if a wavelet coefficient generated by true signal has large magnitude at a finer scale, its ascendants at coarser scales will likely be significant as well. For those coefficients caused by noise, the magnitudes may decay rapidly along the scales. From this observation, it is expected that multiplying the wavelet coefficients at adjacent scales would strengthen the significant structures while diluting noise. Such a property has been exploited for denoising [12]–[13], step estimation [14] and edge detection [15]. The wavelet interscale dependencies have also been represented by
Markov models [16], [17]. Some schemes adopted an interscale and intrascale hybrid model to better estimate noisy wavelet coefficients, such as Liu and Moulin [18] and Portilla et al. [19]. In [19], each coefficient was modeled as the product of a Gaussian random vector and a hidden multiplier variable to include adjacent scales in the conditioning local neighbourhood. The LMMSE denoising schemes in [20] and [21] exploit the wavelet interscale dependencies. Generally, it is seen that performance of interscale LMMSE scheme is wavelet dependent. A rich library of wavelet bases have been constructed and widely used in signal processing, such as Daubechies’ compactly supported orthonormal [22] and biorthogonal wavelets [23]. Our approach is to minimize the wavelet bases searching time. We will achieve this goal by developing an MLP of neural network that smartly chooses the wavelet coefficients. In the recent literature [neup] MLP are deployed for directly denoising the images. Although much better results are reported though this technique.

**Paper Organization:** The rest of the paper is organized as follows. Section II reviews the concept of diffusion wavelets and packets and their application to denoising. Section III we develop OWE based complete denoing model which comprise LMMSE interscale algorithm and for optimal solution we also apply MLP to the absolute scheme. The need, motivation and design characterises are described in detail. In Section IV, experimental results are provided to demonstrate the efficiency for denoising. Finally, conclusions are drawn in Section V.

**Diffusion Wavelets & Packets Review**

Let us consider a semi group \( \{T\} \), associated to a diffusion process (e.g. \( T = e^{c \Delta} \)), here we do not take the Green’s operator, since the latter is not available in the applications we are doing, where the space may be a graph and very little geometrical information is available. We utilize the semi group to induce a multiresolution analysis, interpreting the powers of \( T \) as dilation operators acting on functions, and constructing precise down sampling operators to efficiently represent the multiscale structure. This allows a construction of multiscale scaling functions and wavelets in a very universal setting. The powers of operator \( T \) decreases in rank thus suggesting the compression of the function (and geometric) spaces upon which each power acts. The scheme consists the following steps: First, apply \( T \) to a space of test functions at the finest scale, compress the range via a local orthonormalization procedure, represent \( T \) in the compressed range and compute \( T^i \) on this range, compress its range and orthonormalize, and so on. At scale \( j \) we obtain a compresses representation of \( T^j \), acting on a family of scaling functions spanning the range of \( T^{i2^j} \) basis, and then we apply \( T^{j+1} \), locally orthonormalize and compress the result, thus getting the next coarser subspace. In [24] the complete construction of diffusion wavelets is given but it only provide the mathematical model for it. Fourier analysis and wavelet analysis. The action of a given diffusion semi group on the space of functions on the set is analyzed in a multiresolution fashion, where dyadic powers of the diffusion operator correspond to dilations, and projections correspond to down sampling. The localization of the scaling function constructed allows to reinterpret these operations in function space in a geometric fashion.

Let us take an image contaminated by noise (without loss of generality, we assume additive noise):

\[
I(x) = f(x) + \xi(x)
\]

at this juncture our goal is to remove that noise, resulting in minimal damage to the image. Since in most cases the result end user is human, the criterion for denoising fidelity would be the human visual perception of the result, rather than any of known mathematical criteria, such as minimal mean error (MSE) or minimal maximal difference (minimax).

The most adolescent approach would be performing some kind of low-pass filtering on the image, e.g. by convolution of Gaussian. Perona and Malik [25] claim that low-pass filtering by Gaussian kernel convolution can be equivalently formulated as forward diffusion

\[
\frac{\partial I(x; \tau)}{\partial \tau} = c \Delta I(x; \tau) \quad c = \text{const.}
\]

Where \( \tau \) is the time parameter proportional to the Gaussian standard deviation and \( c \) is the diffusion coefficient. The disadvantage of low-pass filtering or homogenous adaptive diffusion in image enhancement applications is the fact that Gaussian blurring does not respect the natural edges of the image. If we know that a class of functions is well compressed by wavelet packets, then by thresholding the coefficients, we expect to be able to denoise functions from the class (assuming, of course, that the 'noise' is not well compressed by wavelet packets). Efficient and asymptotically optimal denoising algorithms for denoising have been studied by Donoho and Johnstone [26].

**III. OWE & Multiscale Complete LMMSE Model for Denoising**

Orthogonal wavelet transform (OWT) is translation variant due to the down sampling. This will cause some visual artifacts (Gibbs phenomena) in threshold-based denoising [27]. It has been pragmatic that the OWE (undecimated WT or translation-invariant WT in other names) achieves better results in noise reduction and artifacts suppression [28], [29], [30], [31].

The denoising scheme presented in this paper adopts OWE, whose one stage two-dimensional (2-D) decomposition structure is shown in Fig. 1, just as an illustration how it works analytically.
Presume the original signal $f$ is corrupted with additive white Gaussian noise $\varepsilon$

$$g = f + \varepsilon$$

(3)

Where $\varepsilon \in \mathcal{N}(0, \sigma^2)$. Applying the OWE to the noisy signal at scale $j$ yields

$$w_j = x_j + v_j$$

(4)

Where $w_j$ is coefficients at scale $j$, $x_j$ and $v_j$ are the expansions of $f$ and $\varepsilon$ respectively.

Generally we can apply hard and soft thresholding, we take up LMMSE, so LMMSE of $x_j$ is

$$\hat{x}_j = c^j \cdot w_j$$

(5)

Where,

$$c = \frac{\sigma^2 \chi_j}{\sigma^2 \xi_j + \sigma^2 j}.$$  

(6)

It is eminent from [Immse et. zhang] that the wavelet-represented images are similar across scales, especially among the adjacent scales. In wavelet domain, the noise level decrease rapidly along scales, while signal structures are strengthened with scale increasing. Consequently we use coarser scale information to improve finer scale estimation. Assume the input image is decomposed into scales. Generally it can be made that, $j$ scale is strongly correlated with scale $j+1$, but its correlations with $j+2$ and so on will decrease rapidly. These scales would not provide much additional information to improve the estimation of scale $j$. It is clearly evident from fig. 2.

**Fig. 2:** Histograms (solid) of wavelet coefficients of Lena and the associated Gaussian functions (dash) with zero mean and standard deviation $\sigma$. (a) Scale 1(b)Scale 2
From this end the performance of LMMSE method depends on the wavelet filter applied. But apart from applying the conventional method. We follow the approach as shown in fig. 3.

Fig. 3: Diagram showing complete process for denoising

- Noisy Image
- Small Image Patches
- Apply OWE
- Find out LMMSE
- Obtain optimal Wavelet Basis through MLP
- Wavelet Coefficient adjusted
- Apply IWT
- Denoised Patch
- Denoised Image

On the whole process will track the following steps sequentially.
(i) The image is converted to small image patches through simple pre-processing in MATLAB. The reason is simple because it is easy to process image patches in step iv and v.
(ii) We apply Orthogonal Wavelet Expansion as illustrated in previous section with wavelet applied as a diffusion process.
(iii) Find out the LMMSE vector as from eq.6. For more details of LMMSE see [lames].
(iv) Next step is to find out optimally wavlet basis, for that we use the neural network MLP approach as illustrated.

We chose MLPs over other models because of their ability to handle large datasets. A MLP model is shown in fig. 4.

Fig. 4: A graphical representation of a (3,4,2)-MLP.

A multi layer perceptron (MLP) is a nonlinear function that maps vector-valued input via several hidden layers to vector-valued output. For instance, an MLP with two hidden layers can be written as,
\[ f(x)=b_3+W_3\tanh(b_2+W_2\tanh(b_1+W_1x)) \]  (7)
The weight matrices $W_1$, $W_2$, $W_3$ and vector valued bias $b_1$, $b_2$, $b_3$ parameterize the MLP, the function tanh operates component-wise. The architecture of an MLP is defined by the number of hidden layers and by the layer sizes.

The working and performance of the neural network is dependent on the training dataset.

(v) Accordingly, the wavelet coefficient is adjusted. Step 4 &5 will correspond to likely constructing the sparse dictionaries for diffusion wavelets.

(vi) Apply IWT (Inverse Wavelet Transform).

(vii) Denoised Patch is the output.

IV. Experimental Results

Three famous original images (256x256 grayscale) as shown in fig. 5 are taken for experimental purpose from URL Index of/~phao/CIP/Images www.eecs.qmul.ac.uk.

Fig. 5: a, b, c, d indicate example images.

At first glance a random noise is added to the image and then applies the procedure as described in fig. 3. Undoubtedly, the denoised version of the respective images is dependent on the value of sigma (Noise Variance). Let us take $\sigma=10$, at this small value our results are shown in fig. 6.

Fig. 6: Sample image one(a), original image (size 256x256 pixels) at weights =10, middle noisy, top right denoised image

For simulation purpose different .mat files of neural weights, for instance in fig. 5 $\sigma=10$ is occupied. It is evident from the experiment that at lower values of $\sigma$ the proposed scheme is not successfully fitting fit. Hence, the training images is to be increased so that more choice of weights will come up. The next fig. 8 will show this piece of evidence It comprise both noisy and clean image at $\sigma=25$. In figure 9, the value of $\sigma=45$ is put and the neural weights are adjusted according to it.
V. Conclusion
This paper presented a Diffusion Wavelet based novel image denoising scheme for digital camera imaging applications. To fully exploit the spatial and spectral correlations of the images during the denoising process, the training samples from different $\sigma$ were localized by using a supporting MLP structure of neural network. With OWE the wavelet coefficients at the same spatial locations at two adjacent scales are represented as a vector and then LMMSE is applied to the vector. The wavelet interscale dependencies are thus subjugated to improve the signal estimation. The overall performance of the scheme is dependent on the neural weight adjustment guiding principle. The vector find out through LMMSE is further optimized using MLP. MLP help to artificially find out the optimal solution to the problem of finding out the optimal wavelet bases. The proposed scheme also preserves very well the fine structures in the image, which are often smoothed by other denoising schemes.

VI. References
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