An Optimized Derivative Filter for Efficient Edge Detection of Gray Scale Image

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Abstract - Edge Detection is one of the most important tasks in the field of image processing. The edge of an image is a set of connected pixels that lie on the boundary between two regions. Noise is the major problem in the field of image processing. The images are affected by different types of noise. So it is most important task for edge mapping an image in the presence of noise. In image processing edge detection is a technique for mapping edge of an image without removing significant parts of the image contents, such as edges, lines or other details that are important to represent the quality of the image. To acquire a better performance we state another derivative filter that works efficiently for edge mapping an image without removing significant parts of image between. To evaluate the performance we calculate the Signal to Noise Ratio, the Peak Signal to Noise Ratio, the Root Mean Square Error, Image Fidelity and Measuring Similarity between two images. This edge detection operator gives better result with comparison to other existing edge detection operators.

Keywords - Edge Detection, Derivative filter, Gradient Magnitude, Performance Evaluation.

I. INTRODUCTION

Edge detection plays a vital role in image processing for object detection. Edge detection is a type of image segmentation techniques which determines the presence of an edge or line in an image and outlines them in an appropriate way [1]. The main purpose of edge detection is to simplify the image data in order to minimize the amount of data to be processed [2]. Generally, an edge is defined as the pixels of boundary that connect two separate regions with changing image amplitude attributes such as different constant luminance and tristimulus values in an image [1], [3], [4]. The objective of this paper is to find out the relationship between a given pixel’s intensity value and its neighborhood for determining the edge pixels on the image. The finding of edge is very much helpful in solving several problems in the field of Artificial Vision and Image Processing [5]. However, all the edges in an image are not due to the change in intensity values, where parameters like poor focus or refraction can result in edge in an image [6]. The shape of edges in an image depends on different attributes like, lighting conditions, the noise level, types of material and the geometrical and optical properties of the object [7]. Images are often affected by different types of noise such as Salt & pepper noise, Gaussian noise and mixed noise (Impulse and Gaussian) during the transmission, faulty memory location, coherence of waves or timing error. For denoising a corrupted image of Gaussian noise, the Wiener filtering and for Salt & Pepper noise the Median filtering are used as reported by Tukey [8,9]. Sometimes we are to be faced Low Signal to Noise Ratio (SNR), Low Peak Signal to Noise Ratio (PSNR), High Root Mean Square Error (RMSE). But if the SNR is too small or the contrast too low it becomes very difficult to detect anatomical structures because tissue characterization fails. For a visual analysis of images, the clarity of details and the object visibility are important, so high SNR ,PSNR & Low RMSE are required because most of the image segmentation algorithms are very sensitive to noise. In this paper, we state an optimized Edge detection Operator to detect edge map of an image properly that satisfies the image quality criteria. It finds the efficient edge map of an image with improving the SNR, PSNR, RMSE and also other image quality measurement parameters.

II. EDGE DETECTION TECHNIQUES

A. First Derivative Gradient Operators:

The gradient of an Image are determined by computing the partial derivatives $\frac{\partial I}{\partial x}$ and $\frac{\partial I}{\partial y}$ at each pixel location in the Image. We are dealing with digital quantities, so a digital approximation of the partial derivatives over a neighborhood about is required. The Gradient of Image $f(x,y)$ at location $(x,y)$ is calculated by as follows [10]:
\[
Grad(f) = \Delta f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}
\]

Here,  
\( G_x = X \) - Direction derivative  
\( G_y = Y \) - Direction derivative

The Magnitude of Gradient is defined by [10]

\[
|\Delta f| = G(x,y) = \sqrt{G_x^2 + G_y^2} = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}
\]

Here  \( \frac{\partial f}{\partial x} \) and  \( \frac{\partial f}{\partial y} \) partial derivatives of two dimensional function  \( f(x,y) \) along  \( X \) and  \( Y \) axis respectively.

We have used a 3X3 region to denote Image points of an input Image as follows:

| Z1 | Z2 | Z3 |
| Z4 | Z5 | Z6 |
| Z7 | Z8 | Z9 |

**Figure 1:** A 3X3 Region of an Image

Where,  
\( Z1 = f(x-1,y-1), \)  
\( Z2 = f(x-1,y), \)  
\( Z3 = f(x-1,y+1), \)  
\( Z4 = f(x,y-1), \)  
\( Z5 = f(x,y), \)  
\( Z6 = f(x,y+1), \)  
\( Z7 = f(x+1,y-1), \)  
\( Z8 = f(x+1,y), \)  
\( Z9 = f(x+1,y+1) \)

**Figure 2:** a) An edge of an Image Viewed as a 1D Function, b) The derivative of the edge shows a spike at the edge boundary.

1. **Sobel Operator:**

It approximate the magnitude of the gradient at the center of a 3x3 region as [10]

\[
G_x = ( Z7 + 2*Z8 + Z9 ) - ( Z1 + 2*Z2 + Z3 )
\]

\[
G_y = ( Z3 + 2*Z6 + Z9 ) - ( Z1 + 2*Z4 + Z7 )
\]

Here,  
\( G_x = X \) - Direction derivative.  
\( G_y = Y \) - Direction derivative.

The magnitude of the gradient is

\[
|\Delta f| = G(x,y) = \sqrt{G_x^2 + G_y^2}
\]

\[
\begin{array}{ccc}
-1 & -2 & -1 \\
0 & 0 & 0 \\
1 & 2 & 1 \\
\end{array}
\]

**Figure 3:** a) Sobel Operator Mask for Horizontal Direction  
b) Sobel Operator Mask for Vertical Direction

2. **Another Accepted Operator:**

Another Proposed Edge Detection Operator is given in the title of “Filtering Corrupted Image and Edge Detection in Restored Grayscale Image Using Derivative filters” in the Journal of “International Journal of Image Processing (IJIP)” is given by the equations [11]:

\[
G_x = ( Z7 + 3*Z8 + Z9 ) - ( Z1 + 3*Z2 + Z3 )
\]

\[
G_y = ( Z3 + 3*Z6 + Z9 ) - ( Z1 + 3*Z4 + Z7 )
\]
Here \( G_x = X \)-Direction derivative.
\( G_y = Y \)-Direction derivative.

The magnitude of the gradient is
\[
|\Delta f| = G(x,y) = \sqrt{G_x^2 + G_y^2}
\]

(a) \hspace{1cm} (b)

\[
\begin{array}{ccc}
-1 & -3 & -1 \\
0 & 0 & 0 \\
1 & 3 & 1
\end{array}
\hspace{1cm}
\begin{array}{ccc}
-1 & 0 & 1 \\
-3 & 0 & 3 \\
-1 & 0 & 1
\end{array}
\]

Figure 4: a) Another Accepted Operator Mask for Horizontal Direction  
\hspace{1cm} b) Another Accepted Operator Mask for Vertical Direction

B. Second Derivative Gradient Operators:

The foregoing methods of estimating the gradients work best when the gray level transition is quite abrupt, like a step function. As the transition region gets wider it is more advantageous to apply the Second Derivatives. One frequently encountered operator is the Laplacian operator [10], defined as
\[
\Delta^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}
\]

The Laplacian operator is given by the equation:
\[
\Delta^2 f = (Z2 + Z4 + Z6 + Z8) - 4Z5
\]

(a) \hspace{1cm} (b)

\[
\begin{array}{ccc}
0 & -1 & 0 \\
-1 & 4 & -1 \\
0 & -1 & 0
\end{array}
\hspace{1cm}
\begin{array}{ccc}
0 & -1 & 0 \\
-1 & 4 & -1 \\
0 & -1 & 0
\end{array}
\]

Figure 5: a) Laplacian Mask for Horizontal Direction  
\hspace{1cm} b) Laplacian Mask for Vertical Direction

Figure 6: Second Derivative of the signal

III. PROPOSED DERIVATIVE FILTER

Our Proposed Operator is used for edge detection using first derivatives. The magnitudes of the gradient at the center of 3X3 region can be defined as:
\[
G_x = \frac{\partial f}{\partial x} = (0.5*Z7 + Z8 + 0.5*Z9) - (0.5*Z1 + Z2 + 0.5*Z3)
\]
\[
G_y = \frac{\partial f}{\partial y} = (0.5*Z3 + Z6 + 0.5*Z9) - (0.5*Z1 + Z4 + 0.5*Z7)
\]

Here, \( G_x = X \)-Direction derivative.  
\( G_y = Y \)-Direction derivative.

The magnitude of the gradient is
\[
|\Delta f| = G(x,y) = \sqrt{G_x^2 + G_y^2}
\]

\[
\begin{array}{ccc}
-0.5 & -1 & -0.5
\end{array}
\]

Figure 7: a) Proposed Operator Mask for Horizontal Direction
b) Proposed Operator Mask for Vertical Direction

Algorithm:

1. Read the Image I.
2. IF the Image is Noisy THEN
   IF the Type of Noise = ’Gaussian’ THEN
      Filter the Noisy Image I with Winner Filter AND G = ProposedEdgeMapFunction(I)
   ELSE
      Filter the Noisy Image I with Median Filter AND G = ProposedEdgeMapFunction(I)
   END
ELSE
   G = ProposedEdgeMapFunction(I)
3. FINALLY Normalize and Threshold the Gradient Magnitude G to Display the Edge Map.
   1. function [ G ] = ProposedEdgeMapFunction(I)
   2. [m n]=size(I);
   3. w=0.5;
   4. p=1;
   5. for i=2:m-1
      6.    for j=2:n-1
         7.        a=w.*I(i+1,j-1)+p.*I(i+1,j)+w.*I(i+1,j+1);
         8.        b=w.*I(i-1,j-1)+p.*I(i-1,j)+w.*I(i-1,j+1);
         9.        c=w.*I(i-1,j-1)+p.*I(i,j-1)+w.*I(i+1,j-1);
        10.       d=w.*I(i-1,j+1)+p.*I(i,j+1)+w.*I(i+1,j+1);
        11.       G(i,j)=sqrt(((a-b).^2)+((c-d).^2));
      12.    end
     13.  end
   14. end
Figure 9: Flow chart for Degradation, Restoration and Edge Detection of an Image

IV. EVALUATION CRITERIA

To validate the efficiency of this edge Detection Operator we have defined some statistical criteria of image performance. Additionally to subjective visual evaluation, it is desirable to present quantitative measure. The parameters which are used in estimation of performance are SNR, PSNR, RMSE, RMS_SNR, IFy, MSSIM.

A. Signal to Noise Ratio (SNR):

The Signal to Noise Ratio SNR is estimated by the following formula:

\[ SNR = \frac{\sum_{x=1}^{M} \sum_{y=1}^{N} f_1(x,y)^2}{\sum_{x=1}^{M} \sum_{y=1}^{N} (f_2(x,y) - f_1(x,y))^2} \]

Where \( f_1 \) means Original Image and \( f_2 \) means Restored Image. MxN is size of Image and x means row and y means columns.

B. Peak Signal to Noise Ratio (PSNR):

The Peak Signal to noise Ratio PSNR is estimated be the following formula:

\[ PSNR = 10 \log \left( \frac{255^2}{MSE} \right) \]

Where,

\[ MSE = \frac{\sum_{x=1}^{M} \sum_{y=1}^{N} (f_1(x,y) - f_2(x,y))^2}{M \times N} \]

Where \( f_1 \) means Original Image and \( f_2 \) means Restored Image. MxN is size of Image and x means row and y means columns.
C. Root Mean Square Error (RMSE):

The Root Mean Square Error RMSE [5] calculated by the following equation:
\[
RMSE = \sqrt{\text{MSE}}
\]

D. Root Mean Square of Signal to Noise ratio (RMS_SNR):

The Root Mean Square of Signal to Noise Ratio RMS_SNR calculated by the following equation:
\[
\text{RMS}_{\text{SNR}} = \sqrt{\frac{\sum_{x=1}^{M} \sum_{y=1}^{N} f_1(x,y)^2}{\sum_{x=1}^{M} \sum_{y=1}^{N} (f_2(x,y) - f_1(x,y))^2}}
\]

Where \( f_1 \) means Original Image and \( f_2 \) means Restored Image. \( M \times N \) is size of Image and \( x \) means row and \( y \) means columns.

E. Image Fidelity (IFy):

The Image Fidelity [7] is defined by:
\[
IFy = 1 - \frac{1}{\text{SNR}}
\]

F. Measuring Similarity between two image (MSSIM):

MSSIM is used for measuring the similarity between two images i.e. similarity between Original image and Restored image. Higher the MSSIM between Original and filtered image gives lower the noise in filtered image. MSSIM is given by
\[
\text{MSSIM}(x,y) = \frac{1}{M} \sum_{j=1}^{M} \text{SSIM}(x_j, y_j)
\]
\[
\text{SSIM}(x,y) = \frac{(2\mu_x\mu_y+c_1)(2\sigma_{xy}+c_2)}{(\mu_x^2+\mu_y^2+c_1)(\sigma_x^2+\sigma_y^2+c_2)}
\]

Where,
\[
\mu_x = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i}
\]
\[
\sigma_x = \sqrt{\left(\sum_{i=1}^{N} w_i (x_i - \mu_x)^2\right)}
\]
\[
\sigma_{xy} = \sum_{i=1}^{N} w_i (x_i - \mu_x)(y_i - \mu_y)
\]
\[
c_1 = (K_1 L)^2
\]
\[
c_2 = (K_2 L)^2
\]

Where, \( L \) is the range of pixel values (255 for 8-bit grayscale images). And \( K_1 \ll 1 \) is a small constant and also \( K_2 \ll 1 \).

V. EXPERIMENTAL RESULT

To validate the efficiency of our Proposed Derivative Filter, the simulation study has been carried out using MATHLAB Image processing Toolbox. One standard image is selected for simulation study. Firstly, we select the Original Noise free image and then apply to the existing and proposed derivative filter to find edge map. Secondly, we select an contaminated image with Salt & Pepper noise and filtered it using Median filter then we find the edge map by applying to the existing and Proposed derivative filter. Thirdly, we select contaminated image with Gaussian noise and filtered it using Wiener filter and then we find the edge map by applying to the existing and Proposed derivative filter. The subjective and objective comparisons between existing and proposed derivative filters are given bellow.
Table 1: Comparisons of existing and Proposed Derivative Filter for noise free Lena Image

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Laplacian Operator</th>
<th>Sobel Operator</th>
<th>Another Accepted Operator</th>
<th>Proposed Operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>1.4219</td>
<td>2.0230</td>
<td>2.0259</td>
<td>2.0358</td>
</tr>
<tr>
<td>PSNR</td>
<td>52.6255</td>
<td>52.7720</td>
<td>52.7512</td>
<td>52.7976</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.5960</td>
<td>0.5861</td>
<td>0.5875</td>
<td>0.5843</td>
</tr>
<tr>
<td>RMS_SNR</td>
<td>1.1924</td>
<td>1.4223</td>
<td>1.4233</td>
<td>1.4268</td>
</tr>
<tr>
<td>IFy</td>
<td>0.2967</td>
<td>0.5057</td>
<td>0.5064</td>
<td>0.5088</td>
</tr>
<tr>
<td>MSSIM</td>
<td>0.9978</td>
<td>0.9979</td>
<td>0.9979</td>
<td>0.9979</td>
</tr>
</tbody>
</table>

Table 2: Comparisons of existing and Proposed Derivative Filter for Lena Image with Salt & Pepper noise with Noise Density =0.02

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Laplacian Operator</th>
<th>Sobel Operator</th>
<th>Another Accepted Operator</th>
<th>Proposed Operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>1.4160</td>
<td>1.9901</td>
<td>1.9884</td>
<td>2.0043</td>
</tr>
<tr>
<td>PSNR</td>
<td>52.5842</td>
<td>52.6609</td>
<td>52.6336</td>
<td>52.6645</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.5989</td>
<td>0.5936</td>
<td>0.5955</td>
<td>0.5934</td>
</tr>
<tr>
<td>RMS_SNR</td>
<td>1.1899</td>
<td>1.4107</td>
<td>1.4101</td>
<td>1.4157</td>
</tr>
<tr>
<td>IFy</td>
<td>0.2938</td>
<td>0.4975</td>
<td>0.4971</td>
<td>0.5011</td>
</tr>
<tr>
<td>MSSIM</td>
<td>0.9978</td>
<td>0.9979</td>
<td>0.9978</td>
<td>0.9979</td>
</tr>
</tbody>
</table>

Table 3: Comparisons of existing and Proposed Derivative Filter for Lena Image with Gaussian noise with Mean=0 and Standard Deviation=0.02

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Laplacian Operator</th>
<th>Sobel Operator</th>
<th>Another Accepted Operator</th>
<th>Proposed Operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>1.4816</td>
<td>1.8946</td>
<td>1.8797</td>
<td>1.9117</td>
</tr>
<tr>
<td>PSNR</td>
<td>52.8394</td>
<td>52.7630</td>
<td>52.7469</td>
<td>52.7635</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.5815</td>
<td>0.5867</td>
<td>0.5878</td>
<td>0.5866</td>
</tr>
<tr>
<td>RMS_SNR</td>
<td>1.2172</td>
<td>1.3765</td>
<td>1.3710</td>
<td>1.3826</td>
</tr>
<tr>
<td>IFy</td>
<td>0.3251</td>
<td>0.4722</td>
<td>0.4680</td>
<td>0.4769</td>
</tr>
<tr>
<td>MSSIM</td>
<td>0.9979</td>
<td>0.9979</td>
<td>0.9979</td>
<td>0.9979</td>
</tr>
</tbody>
</table>

Visual Comparisons for noise free Original Lena Image of different Edge detection Operators are given below:

![Visual Comparisons for noise free Original Lena Image of different Edge detection Operators](image_url)

Figure 10: a) Original Noise Free Image, b) Edge detection using Laplacian Operator, c) Edge detection using Sobel Operator, d) Edge detection using Another Accepted Operator, e) Edge detection using Our Proposed Operator.

Visual Comparisons for Salt & Pepper noise with Original Lena Image of different Edge detection Operators are given below:

![Visual Comparisons for Salt & Pepper noise with Original Lena Image of different Edge detection Operators](image_url)
VI. CONCLUSION

Edge detection has become a crucial step for detecting a correct object of an Image. We have observed in many theoretical and practical environments that Sobel Operator is better than Roberts and Prewitt operator. So in this paper we just compare our Proposed Operator with Sobel Opearator and Paper Proposed Operator for first derivative filter and also compare the Laplacian Operator for second derivative filter. In the meantime we have applied our Proposed Operator to detect the edge mapping which gives better results than Sobel and other operators. We can make decision by observing the subjective and object comparisons that our Proposed Operator is optimal.

VII. REFERENCES